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仿生纤维增强复合材料层压板的自由振动

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3. 无穷维哈密顿系统及其算法应用教育部重点实验室，内蒙古，呼和浩特 010022)

摘　要：基于 Hamilton 原理和一阶剪切变形板理论，获得了 Winkler-Pasternak 弹性基础上仿生纤维增强复合材料层压板的自由振动控制方程。在四边固支和四边简支的边界条件下，利用微分求积法研究了仿生纤维增强复合材料层压板的无量纲固有频率，讨论了叠层结构、边界条件、宽厚比、无量纲弹性基础刚度以及轴向载荷比对仿生纤维增强复合材料层压板无量纲固有频率的影响。结果表明，固有频率随 Winkler 弹性基础刚度、Pasternak 弹性基础刚度、板宽厚比的增大而增大，随轴向载荷比的增大而减小。

关键词：仿生结构；自由振动；微分求积法；弹性基础；固有频率
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FREE VIBRATION OF BIOMIMETIC FIBER REINFORCED COMPOSITE LAMINATES

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Abstract: Based on Hamilton's principle and the first-order shear deformation plate theory, the free vibration control equation of biomimetic fiber reinforced composite laminates on Winkler Pasternak elastic foundation was proposed. The dimensionless natural frequencies of biomimetic fiber reinforced composite laminates were discussed using the differential quadrature method under the boundary conditions of four fixed and four simply supported edges. The effects of laminated structure, boundary conditions, aspect ratio, dimensionless elastic foundation stiffness, and axial load ratio on the dimensionless natural frequency of biomimetic fiber reinforced composite laminates were examined through numerical examples. The results show that the natural frequency increases with the increase of the Winkler elastic foundation stiffness, the Pasternak elastic foundation stiffness, and the plate width thickness ratio, while decreases with the increase of the axial load ratio.

Key words: biomimetic structure; free vibration; differential quadrature method; elastic foundation; natural frequency

与传统材料相比，纤维增强复合材料具有质量轻、高比强度以及耐腐蚀性等优点，在航空航天、海洋、建筑等多个领域起着越来越重要的作用[1－6]。在亿万年自然进化的过程中，为了应对恶

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劣的生存环境和捕食者的攻击，很多生物进化出优异的力学性能和功能特性[7]。研究发现，虾[8]、甲壳虫[9]、螳螂[10]等诸多天然生物材料内部存在纤维螺旋铺层的微观结构。在层压结构中结合纤维螺旋铺层方式，可以在不损害结构韧性的情况下实现高刚度[10]。同时，与传统的层压板相比，螺旋层压板在平面外强度[11－12]、抗冲击性[12－14]和冲击后压缩强度[12, 15]方面表现出色，因而引起了来自不同领域研究人员的广泛关注。

CHEN 等[16]对仿生螺旋层压板进行了早期研究，发现仿生复合材料的断裂韧性明显高于单向的复合材料。杨立洲等[17]运用修正的非协调广义部分混合元模型分析了纤维铺设角度的变化对变刚度层合板平面内位移场的影响。CHENG 等[9]模仿甲壳类动物骨骼微结构，发现螺旋复合材料层压板在弯曲刚度、强度方面比普通的交叉层压板和准各向同性纤维增强层压板更有优势。MELAIBARI 等[18]通过实验发现，与传统层压板相比，仿生螺旋层压板的强度、失效应变和能量吸收能力均有提高。CHEN 等[19]使用有限元方法发现模仿螳虫的纤维增强 Ti-Al 复合材料比传统的纤维增强复合材料有更高的断裂弯曲应变。HAN 等[20]通过实验研究发现，具有仿生结构的玄武岩纤维增强复合材料的弯曲强度和弯曲模量高于传统的玄武岩纤维增强复合材料。基于一阶剪切变形理论，SHAHBAZTABAR 等[21]研究了双参数弹性地基上对称叠层板在均匀面内载荷作用下的振动问题。NORMAN 等[22]使用有限元模拟了不同纤维取向对复合材料层压板固有频率的影响。根据经典层合板理论，利用 Galerkin 法和最小二乘法，ALTUNSRAY 等[23]讨论了固支或简支对称叠层板在均匀横向载荷作用下的静态弯曲和自由振动。MOHAMED 等[24]采用微分求积法和牛顿迭代法研究了固支或简支边界条件下，螺旋复合材料完美和不完美梁的屈曲、后屈曲和非线性弯曲问题。基于拉普拉斯变换，ALMITANI 等[25]研究了具有线性旋转角的仿生螺旋复合材料梁的屈曲、后屈曲和非线性弯曲问题。MOHAMED 等[26]用微分求积法对纤维螺旋复合材料层合板的静力、屈曲和自由振动问题进行分析，发现层压板序列对弯曲、临界屈曲和固有频率有显著影响。

本文基于 Hamilton 原理和一阶剪切变形板理论，给出了 Winkler-Pasternak 弹性基础上仿生纤维增强复合材料层压板的自由振动控制方程。采用微分求积法对无量纲后的控制方程和边界条件进行离散，得出了层压板的固有频率。分析了叠层结构、边界条件、宽厚比、轴向载荷比、弹性基础刚度对无量纲固有频率的影响。与单向纤维增强复合材料层压板相比，仿生纤维增强复合材料层压板的频率具有可设计性，通过改变叠层结构，可合理调整纤维增强复合材料层压板的频率。

1 理论分析

在三维笛卡尔坐标系下，弹性地基上仿生纤维增强复合材料层合板的几何模型如图 1 所示，由 32 层等厚度的纤维增强复合材料层粘合制成，其厚度为h，长为a，宽为b， xy平面为板的几何中面，K_1, K_2分别表示线性 Winkler 和 Pasternak 弹性基础刚度参数。本文考虑了 5 种复合材料层压板的叠层结构：单向(UD)、准各向同性(QI)、交叉铺层(CP)、线性螺旋(LH)和斐波那契螺旋(FH)。这些叠层结构的堆叠顺序如表 1、图 2 所示。

利用 Hamilton 原理，即：

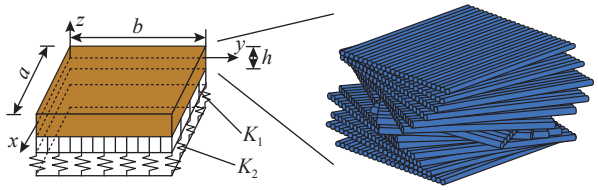


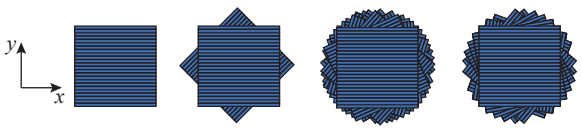
图 1 弹性基础上仿生纤维增强复合材料层压板的几何模型

Fig. 1 Geometric model of biomimetic fiber reinforced composite laminates on elastic foundation

表 1 层压板的叠层结构

Table 1 Layered structure of laminates

名称	N_L	堆叠顺序
UD	32	$[0°/0°\cdots/0°]$
CP	32	$[0°/90°]_{8s}$
QI	32	$[0°/45°/90°/-45°]_{4s}$
LH	32	$[0°/24°\cdots/360°]_s$
FH	32	$[0°/10°/10°/20°/30°/50°/80°/130°/210°/340°/190°/170°/360°/170°/170°/340°]_s$



(a) CP (b) QI (c) LH (d) FH

图 2 叠层结构

Fig. 2 Laminate structures

$$\delta\int_{t_1}^{t_2}(T-U-W)\mathrm{d}t=0 \tag{1}$$

和一阶剪切变形板理论[27]：

$$\begin{cases}\overline{U}(X,Y,Z,t)=U(X,Y,t)+Z\Psi_X(X,Y,t)\\ \overline{V}(X,Y,Z,t)=V(X,Y,t)+Z\Psi_Y(X,Y,t)\\ \overline{W}(X,Y,Z,t)=W(X,Y,t)\end{cases} \tag{2}$$

可得弹性基础上仿生纤维增强复合材料层压板自由振动的控制方程为：

$$N_{XX,X}+N_{XY,Y}=I_A\ddot{U}+I_B\ddot{\psi}_X \tag{3}$$

$$N_{YY,Y}+N_{XY,X}=I_A\ddot{V}+I_B\ddot{\psi}_Y \tag{4}$$

$$\begin{aligned}&Q_{X,X}+Q_{Y,Y}-K_1W+K_2[W_{,XX}+W_{,YY}]+\\&\quad P_{XX}W_{,XX}+P_{YY}W_{,YY}=I_A\ddot{W}\end{aligned} \tag{5}$$

$$M_{XX,X}+M_{XY,Y}-Q_X=I_B\ddot{U}+I_C\ddot{\psi}_X \tag{6}$$

$$M_{XY,X}+M_{YY,Y}-Q_Y=I_B\ddot{V}+I_C\ddot{\psi}_Y \tag{7}$$

式中：“··”表示对时间t的二阶偏导数；U、V和W分别为板的几何中面上沿X轴、Y轴和Z轴的位移；ψ_X、ψ_Y代表几何中面法线关于Y和X轴的转角；M_{XX}、M_{YY}、M_{XY}为作用在几何中面单位长度上的弯矩及扭矩；N_{XX}、N_{YY}、N_{XY}表示法向力；Q_X、Q_Y为剪力；I_A、I_B和I_C为惯性项系数。

纤维增强复合材料层压板的边界条件可表示为：

(1) 四边简支边界条件(SSSS)

$$W=\psi_Y=M_{XX}=0 \qquad X=0,a \tag{8}$$

$$W=\psi_X=M_{YY}=0 \qquad Y=0,b \tag{9}$$

(2) 四边固支边界条件(CCCC)

$$W=\psi_X=\psi_Y=0 \qquad X=0,a;Y=0,b \tag{10}$$

在本文中，$X=a$是可移的，$X=0,Y=0,b$是不可移的。相应的边界条件为：

$$U=V=0,\quad X=0 \tag{11}$$

$$N_{XX}=V=0,\quad X=a \tag{12}$$

$$N_{XY}=V=0,\quad Y=0,b \tag{13}$$

引入无量纲参数：

$$\begin{aligned}&(x,y)=(X/a,Y/b),(u,v,w)=(U,V,W)/h,\\&(\varphi_x,\varphi_y)=(\psi_X,\psi_Y),(\lambda_1,\lambda_2)=a/(h,b),\\&(a_{ij},b_{ij},d_{ij})=(A_{ij},B_{ij}/h,D_{ij}/h^2)/A_{110},\\&(k_1,k_2)=(a^2K_1,K_2)/A_{110},(p_{xx},p_{yy})=(P_{XX},P_{YY})/A_{110},\\&\tau=t\sqrt{A_{110}/I_{00}}/a,(I_0,I_1,I_2)=(I_A,I_B/h,I_C/h^2)/I_{00},\\&A_{110}=E_2h^3/a^2,I_{00}=\rho h.\end{aligned}$$

将上述无量纲参数代入式(3)~式(7)中，得到量纲归一化后的控制方程和边界条件：

$$\begin{aligned}&a_{11}u_{,xx}+2a_{16}\lambda_2u_{,xy}+a_{66}\lambda_2^2u_{,yy}+(a_{12}+a_{66})\lambda_2v_{,xy}+\\&\quad a_{16}v_{,xx}+a_{26}\lambda_2v_{,yy}+b_{11}\varphi_{x,xx}+2b_{16}\lambda_2\varphi_{x,xy}+\\&\quad b_{66}\lambda_2^2\varphi_{x,yy}+(b_{12}+b_{66})\lambda_2\varphi_{y,xy}+b_{16}\varphi_{y,xx}+\\&\quad b_{26}\lambda_2\varphi_{y,yy}=I_0\ddot{u}+I_1\ddot{\varphi}_x\end{aligned} \tag{14}$$

$$\begin{aligned}&a_{16}u_{,xx}+(a_{12}+a_{66})\lambda_2u_{,xy}+a_{26}\lambda_2^2u_{,yy}+a_{66}v_{,xx}+\\&\quad 2a_{26}\lambda_2v_{,xy}+a_{22}\lambda_2v_{,yy}+b_{16}\varphi_{x,xx}+b_{26}\lambda_2^2\varphi_{x,yy}+\\&\quad (b_{12}+b_{66})\lambda_2\varphi_{x,xy}+b_{66}\varphi_{y,xx}+2b_{26}\lambda_2\varphi_{y,xy}+\\&\quad b_{22}\lambda_2\varphi_{y,yy}=I_0\ddot{v}+I_1\ddot{\varphi}_y\end{aligned} \tag{15}$$

$$\begin{aligned}&\kappa a_{55}w_{,xx}+2\kappa a_{45}\lambda_2w_{,xy}+\kappa a_{44}\lambda_2^2w_{,yy}+\kappa a_{55}\lambda_1\varphi_{x,x}+\\&\quad \kappa a_{45}\lambda_1\lambda_2\varphi_{x,y}+\kappa a_{45}\lambda_1\varphi_{y,x}+\kappa a_{44}\lambda_1\lambda_2\varphi_{y,y}+\\&\quad p_{xx}w_{,xx}+p_{yy}\lambda_2^2w_{,yy}-k_1w+k_2w_{,xx}+k_2\lambda_2^2w_{,yy}=\\&\quad I_0\ddot{w}\end{aligned} \tag{16}$$

$$\begin{aligned}&b_{11}u_{,xx}+2b_{16}\lambda_2u_{,xy}+b_{66}\lambda_2^2u_{,yy}+(b_{12}+b_{66})\lambda_2v_{,xy}+\\&\quad b_{16}v_{,xx}+b_{26}\lambda_2^2v_{,yy}+d_{11}\varphi_{x,xx}+2d_{16}\lambda_2\varphi_{x,xy}+\\&\quad d_{66}\lambda_2^2\varphi_{x,yy}+(d_{12}+d_{66})\lambda_2\varphi_{y,xy}+d_{16}\varphi_{y,xx}+\\&\quad d_{26}\lambda_2^2\varphi_{y,yy}-\kappa a_{45}\lambda_1\lambda_2w_{,y}-\kappa a_{55}\lambda_1w_{,x}-\\&\quad \kappa a_{45}\lambda_2\varphi_y-\kappa a_{55}\lambda_2\varphi_x=I_1\ddot{u}+I_2\ddot{\varphi}_x\end{aligned} \tag{17}$$

$$\begin{aligned}&b_{12}u_{,xx}+(b_{12}+b_{66})\lambda_2u_{,xy}+b_{26}\lambda_2^2u_{,yy}+2b_{26}\lambda_2v_{,xy}+\\&\quad b_{66}v_{,xx}+b_{22}\lambda_2^2v_{,yy}+(d_{12}+d_{66})\lambda_2\varphi_{x,xy}+\\&\quad d_{16}\varphi_{x,xx}+d_{26}\lambda_2^2\varphi_{x,yy}+2d_{26}\lambda_2\varphi_{y,xy}+d_{66}\varphi_{y,xx}+\\&\quad d_{22}\lambda_2^2\varphi_{y,yy}-\kappa a_{44}\lambda_1\lambda_2w_{,y}-\kappa a_{45}\lambda_1w_{,x}-\\&\quad \kappa a_{44}\lambda_2\varphi_y-\kappa a_{45}\lambda_2\varphi_x=I_1\ddot{v}+I_2\ddot{\varphi}_y\end{aligned} \tag{18}$$

简支边界条件(SSSS)：

$$\begin{aligned}&x=0:\\&\quad u=v=w=\varphi_y=0,\\&\quad b_{11}u_{,x}+b_{12}\lambda_2v_{,y}+b_{16}(\lambda_2u_{,y}+v_{,x})+d_{11}\varphi_{x,x}+\\&\qquad d_{12}\lambda_2\varphi_{y,y}+d_{16}(\lambda_2\varphi_{x,y}+\varphi_{y,x})=0,\\&x=1:\\&\quad v=w=\varphi_x=0,\\&\quad a_{11}u_{,x}+a_{12}\lambda_2v_{,y}+a_{16}(\lambda_2u_{,y}+v_{,x})+b_{11}\varphi_{x,x}+\\&\qquad b_{12}\lambda_2\varphi_{y,y}+b_{16}(\lambda_2\varphi_{x,y}+\varphi_{y,x})=0,\\&\quad b_{11}u_{,x}+b_{12}\lambda_2v_{,y}+b_{16}(\lambda_2u_{,y}+v_{,x})+d_{11}\varphi_{x,x}+\\&\qquad d_{12}\lambda_2\varphi_{y,y}+d_{16}(\lambda_2\varphi_{x,y}+\varphi_{y,x})=0,\\&y=0,1:\\&\quad v=w=\varphi_x=0,\\&\quad b_{12}u_{,x}+b_{22}\lambda_2v_{,y}+b_{26}(\lambda_2u_{,y}+v_{,x})+d_{12}\varphi_{x,x}+\\&\qquad d_{22}\lambda_2\varphi_{y,y}+d_{26}(\lambda_2\varphi_{x,y}+\varphi_{y,x})=0,\\&\quad a_{16}u_{,x}+a_{26}\lambda_2v_{,y}+a_{66}(\lambda_2u_{,y}+v_{,x})+b_{16}\varphi_{x,x}+\\&\qquad b_{26}\lambda_2\varphi_{y,y}+b_{66}(\lambda_2\varphi_{x,y}+\varphi_{y,x})=0,\end{aligned} \tag{19}$$

固支边界条件(CCCC)：

$$\begin{aligned}&x=0:\\&\quad u=v=w=\varphi_x=\varphi_y=0,\\&x=1:\\&\quad v=w=\varphi_x=\varphi_y=0,\\&\quad a_{11}u_{,x}+a_{12}\lambda_2v_{,y}+a_{16}(\lambda_2u_{,y}+v_{,x})+b_{11}\varphi_{x,x}+\\&\qquad b_{12}\lambda_2\varphi_{y,y}+b_{16}(\lambda_2\varphi_{x,y}+\varphi_{y,x})=0,\\&y=0,1:\\&\quad v=w=\varphi_x=\varphi_y=0,\end{aligned}$$

$y=0,1$:

$$v=w=\varphi_x=\varphi_y=0,$$
$$a_{16}u_{,x}+a_{26}\lambda_2 v_{,y}+a_{66}(\lambda_2 u_{,y}+v_{,x})+b_{16}\varphi_{x,x}+b_{26}\lambda_2\varphi_{y,y}+b_{66}(\lambda_2\varphi_{x,y}+\varphi_{y,x})=0, \tag{20}$$

2 求解过程

对于式 (14)~式 (18) 所示的仿生纤维增强复合材料层压板的控制方程，本文采用微分求积法进行求解。根据微分求积法的离散化法则，位移分量$u,v,w,\varphi_x,\varphi_y$及其在网格点$(x_i,y_j)$关于$x$和$y$的第$k_1$和$k_2$阶导数近似为：

$$\{u,v,w,\varphi_x,\varphi_y\}=\sum_{m=1}^{N_1}\sum_{n=1}^{N_2}l_m(x)l_n(y)\{u_{mn},v_{mn},w_{mn},\varphi_{xmn},\varphi_{ymn}\} \tag{21}$$

$$\frac{\partial^{k_1}}{\partial x^{k_1}}\frac{\partial^{k_2}}{\partial y^{k_2}}\{u,v,w,\varphi_x,\varphi_y\}|_{x=x_i,y=y_j}=\sum_{m=1}^{N_1}\sum_{n=1}^{N_2}C_{im}^{(k_1)}C_{jn}^{(k_2)}\{u_{mn},v_{mn},w_{mn},\varphi_{xmn},\varphi_{ymn}\} \tag{22}$$

式中：$\{u,v,w,\varphi_x,\varphi_y\}$在$(x,y)=(x_m,y_n)$处的取值为$\{u_{mn},v_{mn},w_{mn},\varphi_{xmn},\varphi_{ymn}\}$；$l_m(x),l_n(y)$为拉格朗日插值多项式；$C_{im}^{(k_1)},C_{jn}^{(k_2)}$是关于节点的$k_1,k_2$阶导数的权系数；$N_1,N_2$分别为沿$x$轴、y轴分布的节点总数。

选用非均匀的 Chebyshev-Gauss-Lobatto 分布划分网格，其坐标可表示为：

$$x_i=\frac{1}{2}\left[1-\cos\frac{\pi(i-1)}{N_1-1}\right],\ i=1,2,\cdots,N_1 \tag{23}$$

$$y_j=\frac{1}{2}\left[1-\cos\frac{\pi(j-1)}{N_2-1}\right],\ j=1,2,\cdots,N_2 \tag{24}$$

将式 (21) 和式 (22) 代入式 (14)~式 (18)，可得：

$$(\boldsymbol{K}_{\mathrm{L}}-P\boldsymbol{K}_{\mathrm{P}})\boldsymbol{d}+\boldsymbol{M}\ddot{\boldsymbol{d}}=0 \tag{25}$$

系数矩阵$\boldsymbol{K}_{\mathrm{L}},\boldsymbol{K}_{\mathrm{P}}$为刚度矩阵，$\boldsymbol{M}$为质量矩阵，$\boldsymbol{K}_{\mathrm{L}},\boldsymbol{K}_{\mathrm{P}},\boldsymbol{M}$是$5N_1N_2\times5N_1N_2$的矩阵，$\boldsymbol{d}$是位移向量$u_{ij},v_{ij},w_{ij},\varphi_{xij},\varphi_{yij}(i=1,2,\cdots N_1;j=1,2,\cdots N_2)$。

由式 (25) 可得板的无量纲临界屈曲载荷P_{cr}：

$$(\boldsymbol{K}_{\mathrm{L}}-P_{\mathrm{cr}}\boldsymbol{K}_{\mathrm{P}})\boldsymbol{d}=0 \tag{26}$$

令$\boldsymbol{d}=\boldsymbol{d}^*e^{\mathrm{i}\omega\tau}$，由式 (25) 可得：

$$(\boldsymbol{K}_{\mathrm{L}}-P\boldsymbol{K}_{\mathrm{P}}-\omega^2\boldsymbol{M})\boldsymbol{d}^*=0 \tag{27}$$

式中，ω为无量纲固有频率，通过求解式 (27) 可得仿生纤维增强复合材料层压板的无量纲固有频率。

3 数值算例分析

下面分析无量纲弹性基础刚度、层压板的叠层结构、边界条件、宽厚比、轴向载荷比等参数对仿生纤维增强复合材料层压板自由振动的影响。在所有计算过程中，材料物性参数取$E_1/E_2=10$，$G_{12}=G_{13}=0.5E_2,G_{23}=0.2E_2,\upsilon_{12}=0.25$，$\rho=1000\ \mathrm{kg/m^3}$。

3.1 对比验证

为了验证计算方法的正确性，表 2 和表 3 列出了采用微分求积法计算得到的无量纲固有频率，并与 REDDY[27] 基于一阶剪切变形理论得到的解析解和 XIANG 等[28] 通过逆多元二次径向基函数的无网格配置法得到的数值解进行对比，结果具有很好的一致性。

表 2 对称简支方板的无量纲固有频率

$[0°/90°/90°/0°]$

$(E_1/E_2=40,G_{12}=G_{13}=0.6E_2,G_{23}=0.5E_2)$

Table 2 The dimensionless natural frequency of symmetric simply supported square plates$[0°/90°/90°/0°]$ $(E_1/E_2=40,G_{12}=G_{13}=0.6E_2,G_{23}=0.5E_2)$

方法	a/h		
	4	10	20
文献[27]	9.497	15.270	17.668
文献[28]	9.496	15.271	17.707
本文	9.395	15.143	17.660

表 3 非对称简支方板的无量纲固有频率

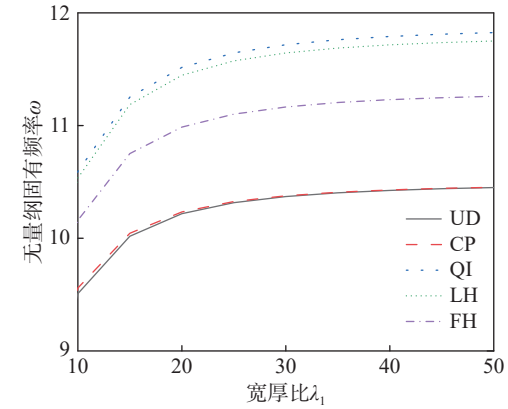
Table 3 The dimensionless natural frequency of asymmetric simply supported square plates

	a/h	方法	E_1/E_2		
			10	25	40
$[0°/90°]$	10	文献[27]	7.454	8.900	10.027
		本文	7.454	8.900	10.027
	20	文献[27]	7.802	9.474	10.840
		本文	7.802	9.475	10.840
	100	文献[27]	7.926	9.687	11.150
		本文	7.926	9.687	11.150
$[0°/90°]_4$	10	文献[27]	9.450	12.628	14.560
		本文	9.450	12.628	14.562
	20	文献[27]	10.102	14.241	17.169
		本文	10.102	14.241	17.169
	100	文献[27]	10.344	14.912	18.365
		本文	10.344	14.912	18.365

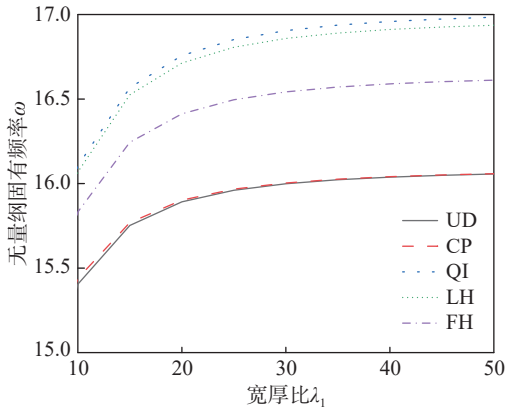
3.2 自由振动

图 3~图 4 给出了当$\lambda_2=1$时，不同宽厚比及弹性基础刚度下 SSSS 和 CCCC 板的无量纲固有频率。结果表明，无量纲固有频率随 Winkler、Pasternak 弹性基础刚度、板的宽厚比的增大而增大。对于 SSSS 板，QI 和 LH 结构的无量纲固有频率较高，UD 和 CP 结构的频率较低。在 CCCC 边

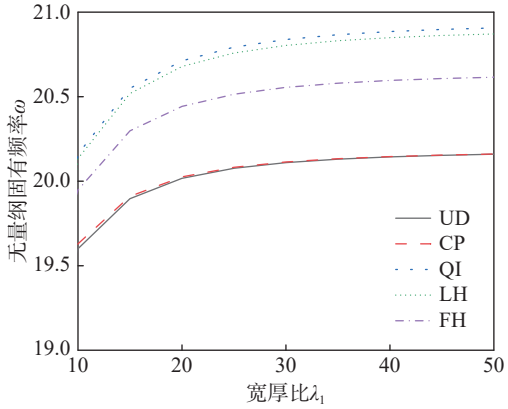
界条件下，CP 结构具有较高的频率，FH 结构具有较低的频率。



(a) 弹性基础刚度(k_1, k_2)=(0, 0)



(b) 弹性基础刚度(k_1, k_2)=(50, 5)



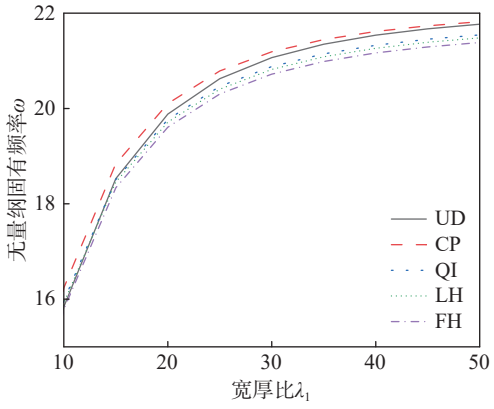
(c) 弹性基础刚度(k_1, k_2)=(100, 10)

图 3　不同宽厚比(λ_1)及弹性基础刚度下 SSSS 板的无量纲固有频率

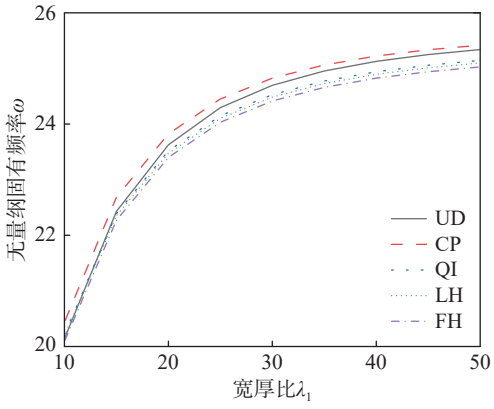
Fig. 3　Dimensionless natural frequencies of SSSS plates under different aspect ratios(λ_1) and elastic foundation stiffness

图 5 是不同叠层结构 CCCC 板的前三阶振型，可以注意到，叠层结构对模态振型的影响较小，对无量纲固有频率的影响较大。

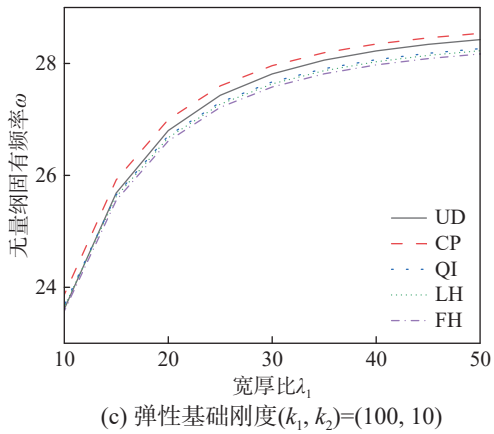
在沿x轴或y轴施加载荷的情况下 (如图 6)，图 7~图 11 考虑了参数为 $\lambda_1=10, \lambda_2=1, (k_1, k_2)=(50, 5)$时，SSSS 板的无量纲固有频率$\omega$与平面内载荷比 ($P_0=P/P_{cr}$)的关系曲线。由图可见：当$\lambda_1, \lambda_2, k_1, k_2$一定时，对于所有类型的压缩轴向力，无论叠层结构如何变化，无量纲固有频率都会随着轴向载荷比的增大而减小。可以观察到，在类型为$(0, -P_0)$的压缩轴向力作用下产生的无量纲固有频率高于其他两种类型。同时，对于不同的叠层结构和压缩轴向力类型，无量纲固有频率的变



(a) 弹性基础刚度(k_1, k_2)=(0, 0)



(b) 弹性基础刚度(k_1, k_2)=(50, 5)



(c) 弹性基础刚度(k_1, k_2)=(100, 10)

图 4　不同宽厚比(λ_1)及弹性基础刚度下 CCCC 板的无量纲固有频率

Fig. 4　Dimensionless natural frequencies of CCCC plates under different aspect ratios(λ_1) and elastic foundation stiffness

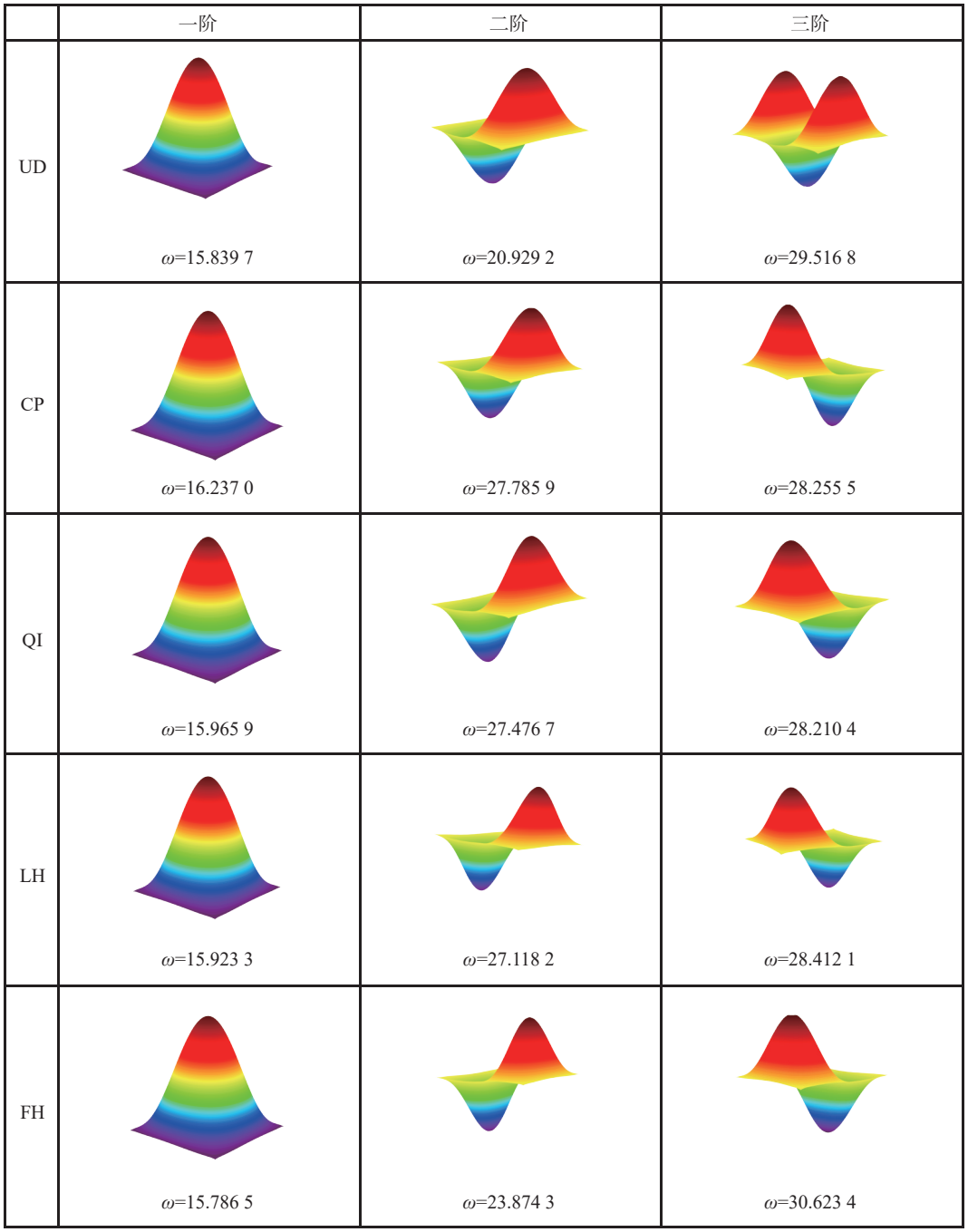


图 5 不同叠层结构的 CCCC 方板的前三阶振型

Fig. 5 The first three vibration modes of CCCC square plate with different stacked structures

化是不同的。

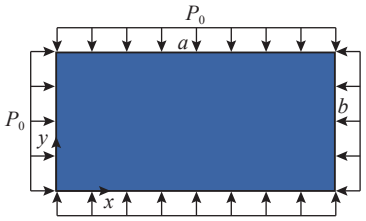
4 结论

本文基于 Hamiton 原理和一阶剪切变形板理论给出了 Winkler-Pasternak 弹性基础上仿生纤维增强复合材料层压板的自由振动控制方程，然后采用微分求积法对无量纲后的控制方程和边界条件进行离散，得到了层压板的固有频率。分析了叠层结构、边界条件、宽厚比、轴向载荷比、弹性基础刚度对无量纲固有频率的影响。结果表明：

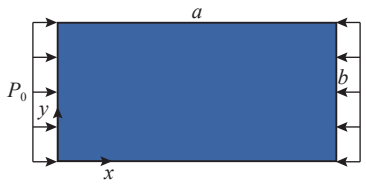
(1) 在弹性基础刚度不变的情况下，仿生纤维增强复合材料层压板的无量纲固有频率ω随着宽厚比λ_1的增大而增大，且不同叠层结构的无量纲固有频率不同，SSSS 板 QI 结构的频率较高，CCCC 板 CP 结构的频率较高。

(2) 对于简支和固支两种边界条件，仿生纤维增强复合材料层压板的无量纲固有频率ω随着无量纲弹性基础刚度的增大而增大。

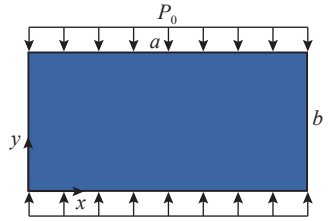
(3) 当$\lambda_1,\lambda_2,k_1,k_2$一定时，对于所有类型的压缩轴向力，无论叠层结构如何变化，无量纲固有频



(a) $(-P_0, -P_0)$



(b) $(-P_0, 0)$



(c) $(0, -P_0)$

图 6　不同类型的载荷方式

Fig. 6　Various types of load modes

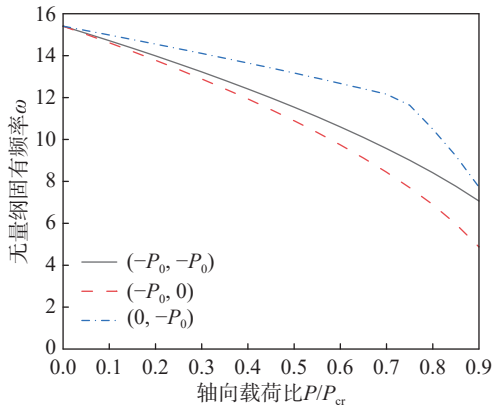


图 7　UD 结构下，轴向载荷比对频率的影响

Fig. 7　Effect of axial load ratio on frequency under UD structure

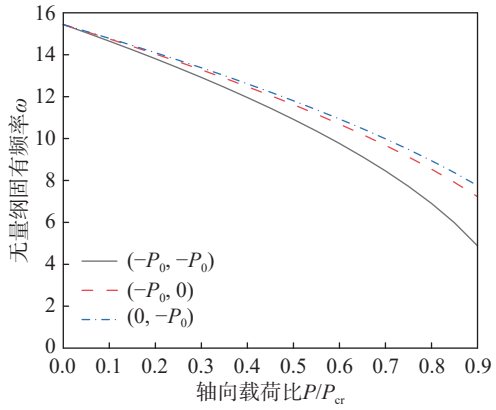


图 8　CP 结构下，轴向载荷比对频率的影响

Fig. 8　Effect of axial load ratio on frequency under CP structure

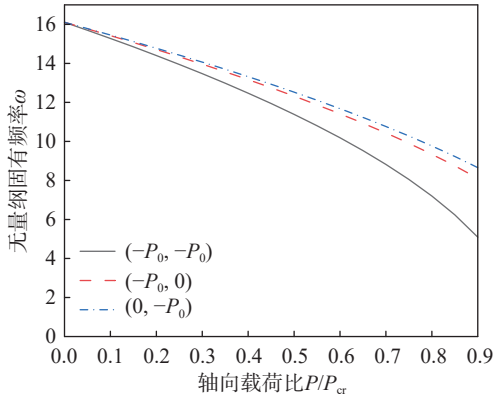


图 9　QI 结构下，轴向载荷比对频率的影响

Fig. 9　Effect of axial load ratio on frequency under QI structure

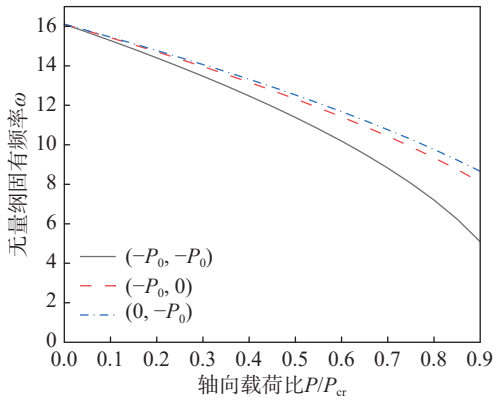


图 10　LH 结构下，轴向载荷比对频率的影响

Fig. 10　Effect of axial load ratio on frequency under LH structure

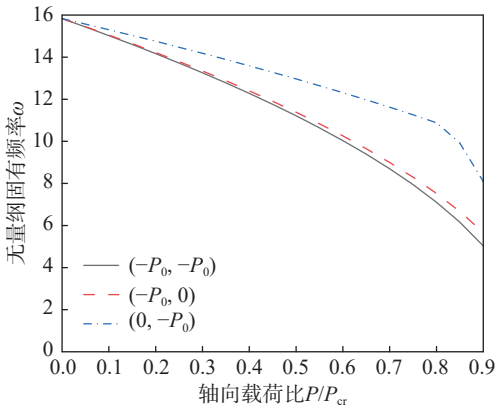


图 11　FH 结构下，轴向载荷比对频率的影响

Fig. 11　Effect of axial load ratio on frequency under FH structure

率都会随着轴向载荷比的增大而减小。

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